

ORIE 3120 Homework 5

Due March 11, 2020 at 11:59PM on Gradescope

Question 1 (SQL + Newsvendor with Discrete Demand)

We will use SQL to compute the optimal order quantity in the newsvendor problem for a discrete demand distribution. This is less computationally efficient than the bisection algorithm we discussed in lecture, but if your data is already in an SQL database then it is really convenient to be able to compute everything in SQL. Use the database Newsvendor.sqlite (posted on the course site), which contains two tables.

The first table, called Demand, contains a single field with historical data on demand, where each record represents the demand observed on a particular day in history. The second table, called WholeNumbers, contains a field N with one record with each of the whole numbers between 0 and 20.

To do our calculation, recall from lecture that the optimal order quantity Q is an integer such that $P(D \leq Q) \geq q$ and $P(D \leq Q - 1) < q$ where $q = c_u / (c_u + c_o)$. Another equivalent way of saying this is that the optimal order quantity Q is the smallest Q with $P(D \leq Q) \geq q$. We will write a sequence of queries that will calculate this smallest Q .

- Calculate $P(D \leq 5)$ using the data in the database. Explain how you calculated this value and include your query in your submission.
- Write a query called Q01 that produces the following results. The first field is N from the WholeNumbers table. The second field is the number of records in Demand that have $D \leq N$.

N	NumRecordsWithDLessThanOrEqualToN
0	16
1	31
2	44
3	53
4	70
5	78
6	97
7	108
8	118
9	134
10	146
11	157
12	173
13	182
14	192
15	211
16	216
17	229
18	238
19	246

- Write a second query, called Q02, that returns only those records from view Q01 with $P(D \leq N) \geq q$, where q is hard-coded to 0.5. To write this query, you will need to understand how to calculate $P(D \leq N)$ from the field NumRecordsWithDLessThanOrEqualToN from Q01.

Note that NumRecordsWithDLessThanOrEqualToN is an integer, and if you divide an integer by another integer that is larger than it, SQL will give you a zero. While there is a way to divide integers so that this doesn't happen, instead for this question just write your query in a way that you don't have to divide an integer by another integer.

You should get a query result that looks like this:

N	NumRecordsWithDLessThanOrEqualToN
9	134
10	146
11	157
12	173
13	182
14	192
15	211
16	216
17	229
18	238
19	246

- (d) Write a third query, called Q03, that returns the record in Q02 with the smallest value for N. This is the optimal Q . It should produce the single record below. Based on this result, the optimal value when $q = 0.5$ is $Q = 9$.

N	NumRecordsWithDLessThanOrEqualToN
9	134

- (e) Write a single query that uses only the tables Demand and WholeNumbers and produces the same result as Q03. Then change this query to calculate the optimal Q for $q = 0.4$. Include your query and the new optimal value it computes in your submission.

Note: Queries must be copied and pasted into your submission pdf. Screenshots of them are insufficient.

Question 2 (Newsvendor with Normal Demand)

Yvette Brown runs the coffee shop in Duffield Hall. She orders bagels from Ithaca Bakery to sell each day. The cost of each bagel is \$0.54, and each sells for \$1.08. Bagels that do not sell by the end of the day are stale, so they are sold to a bagel chip manufacturer for \$0.24 each. The daily demand is stationary, random, independent, and normally distributed, with mean of 45 and a standard deviation of 9.

- (a) Find the optimal order quantity.
- (b) Find the expected annual underage/overage cost if the shop is open 180 days per year.
- (c) Yvette is replaced by Rex Price. Rex decides to order 63 bagels per day. Find the additional annual cost of Rex's policy.

Question 3 (Q,R Policies with Normal Demand)

Fry's electronics, a large electronics retailer, purchases 1TB hard disks and sells them to customers. The store operates on a continuous review inventory system.

- Weekly demand for disk drives is independent and normally distributed with a mean of 50 and a standard deviation of 15.
 - Lead time is 2 weeks
 - Holding cost for a disk drive is \$1/unit/week
 - Ordering costs are \$400.00/order
 - Desired fill rate is 98.0%
- (a) What order quantity and reorder point, (Q, R), should the distribution center use?
 - (b) What order quantity and reorder point, (Q, R), should the distribution center use if, instead of a fill rate of 98%, a penalty cost of $p = \$10/\text{unit}$ is used?

Question 4 (Preparation for Excel Simulation of Q,R policies)

- (a) Suppose that demand over the course of a week is normally distributed with mean μ and variance σ^2 . Also suppose that this demand is the sum of independent normally distributed demands on each day of mean a and variance b^2 . Express a and b in terms of μ and σ^2 .
- (b) What is the expected average cost per week when operating a Q,R policy in the inventory problem in Question 3(b), when $Q = 216$, $R = 104$, and Fry's pays $c = \$75$ per disk drive.

Question 5 (Excel Simulation of Q,R Policies)

Create an Excel spreadsheet like the one in the screenshot below to simulate a Q,R policy using the parameters in Question 4(b). We give more details and hints on how to create this spreadsheet below. The value you calculate for the average cost per day (in cell E7) should be close to the value you get in Question 4(b) above, but won't be exactly the same because of randomness and assumptions made in the theory but violated in the simulation that (1) we place an order immediately when inventory falls below R; and (2) negative inventory incurs negative holding costs. Note that the only values manually entered are in cells B1:B9. Cells in rows 11 and onward use these parameters to simulate the inventory policy, and the values in cells E1:E7 are calculated by aggregating and summarizing the simulation data.

	A	B	C	D	E	F	G	H	I	J	
1	Penalty Cost	10		Average Holding Cost Per Day	16.01						
2	Q	216		Average Stockouts Per Day	0.34						
3	R	104		Average Demand Per Day	7.18						
4	Tau (days)	14		Fill Rate	0.95						
5	h (\$ per day)	0.14		Average Order Cost Per Day	551.12						
6	K (\$)	400		Average Penalty Cost Per Day	3.44						
7	mu	7.142857143		Average Cost Per Day	570.57						
8	sigma	5.669467095									
9	c (\$)	75									
							Is order en-route? (If yes, show days till it arrives. If not, show a negative number)	Holding Cost	Stockouts	Order Cost	
10		Day	Inventory at Start of Day	Demand	Inventory at End of Day	Place Order? [1=Yes, 0=No]					
11		1	216.0		10.9	205.1	FALSE	-1	30.9	0	0
12		2	205.1		3.0	202.1	FALSE	-2	29.3	0	0
13		3	202.1		7.8	194.4	FALSE	-3	28.9	0	0
14		4	194.4		7.6	186.8	FALSE	-4	27.8	0	0
15		5	186.8		1.2	185.6	FALSE	-5	26.7	0	0
16		6	185.6		4.8	180.9	FALSE	-6	26.5	0	0
17		7	180.9		17.2	163.7	FALSE	-7	25.8	0	0
18		8	163.7		3.2	160.5	FALSE	-8	23.4	0	0
19		9	160.5		6.9	153.6	FALSE	-9	22.9	0	0
20		10	153.6		7.7	145.9	FALSE	-10	21.9	0	0
21		11	145.9		5.9	140.0	FALSE	-11	20.8	0	0
22		12	140.0		9.2	130.9	FALSE	-12	20.0	0	0
23		13	130.9		-1.6	132.4	FALSE	-13	18.7	0	0
24		14	132.4		3.8	128.6	FALSE	-14	18.9	0	0
25		15	128.6		2.4	126.2	FALSE	-15	18.4	0	0
26		16	126.2		0.5	125.7	FALSE	-16	18.0	0	0
27		17	125.7		15.3	110.3	FALSE	-17	18.0	0	0
28		18	110.3		0.8	109.6	FALSE	-18	15.8	0	0
29		19	109.6		9.1	100.5	TRUE	14	15.7	0	16600
30		20	100.5		7.9	92.5	FALSE	13	14.4	0	0
31		21	92.5		16.3	76.2	FALSE	12	13.2	0	0
32		22	76.2		9.4	66.8	FALSE	11	10.9	0	0
33		23	66.8		17.2	49.7	FALSE	10	9.5	0	0
34		24	49.7		10.4	39.3	FALSE	9	7.1	0	0
35		25	39.3		4.6	34.7	FALSE	8	5.6	0	0
36		26	34.7		7.3	27.3	FALSE	7	5.0	0	0
37		27	27.3		14.0	13.3	FALSE	6	3.9	0	0
38		28	13.3		13.7	-0.4	FALSE	5	1.9	0.4142329	0
39		29	-0.4		8.6	-9.0	FALSE	4	-0.1	8.58424279	0
40		30	-9.0		13.0	-22.0	FALSE	3	-1.3	12.9850858	0
41		31	-22.0		-1.9	-20.1	FALSE	2	-3.1	0	0
42		32	-20.1		7.6	-27.7	FALSE	1	-2.9	7.62415432	0
43		33	-27.7		4.2	-31.9	FALSE	0	-4.0	4.23031092	0

Include TWO screenshots in your submission

1. A screenshot of the top of your completed spreadsheet (similar to the screenshot above)
2. A screenshot of the formulas in your spreadsheet. You can show formulas in Excel by clicking on the "Show Formulas" button in the "Formula Auditing" section of the "Formulas Tab". In Google Sheets, click view → show formulas. Be sure to adjust the column widths so they are not cut off.

Hints for Question 5:

- The value for cells B7 and B8 are derived from the formula you created in Question 4(a).
- Values in cells E1, E2, E3, and E5 are calculated by averaging values in individual columns in the table starting in row 11. Values in cells E4, E6, and E7 are calculated using formulas that reference cells E1, E2, E3, and E5 (i.e. the averages).
- To simulate demand in column D, use $\text{NORM.INV}(\text{RAND}(), \text{mean}, \text{sigma})$. This will generate a normal random variable whose mean is μ and standard deviation is σ .
- The values in each row of the table starting in row 11 columns C through G are computed from left-to-right and top to bottom, with each cell using values from the cells to the left of it and in some cases from the row above. These rows implement the logic of a Q,R policy.
 - We start day 1 with Q units of inventory. In each subsequent day, inventory at the start of the day is the sum of the inventory at the end of the previous day and any order that arrived on the previous day (which is indicated by a 0 in column G).
 - We place an order when no order is currently on its way, and when the inventory at the end of the day is below R.
 - We keep track of whether an order is on its way (“en-route”) and how long it will take to get here with a number in column G. When this number is positive, this is the number of days until the order arrives. When the number is negative, it is the number of days since the order has arrived.
- The formulas used in row 12 should be constructed so that they can be filled/copied down for all subsequent rows. Some formulas in row 11 may be slightly different because some initial values are given.
- We calculate columns H, I and J from columns C, D and F (and the parameters in B2:B9).
- Use 5000 days.
- The screenshot sets the number of decimal places to show using Format -> Cells -> Number. You do not need to do this.